The hybrid was designed with a 39-Ω system and employed near-optimum transmission-line tapers [2] to 50-Ω lines.

Parasitic junction reactances have a significant effect on the characteristics of the hybrid. Since an analytical description of the complex junctions found would be very difficult to obtain, a superposition of equivalent circuits which have been analytically described elsewhere was employed. Leighton and Milnes [3] have shown that an asymmetrical T junction may be approximated by a symmetrical T junction for microstrip parasitic calculations provided W_1^* is approximately equal to W_2^*. The current authors have assumed that junctions of the form shown in Fig. 2 may be considered as a superposition of two T junctions, again provided W_1^* is approximately equal to W_2^*. In the hybrid structure constructed in microstrip, W_2^* is found to be much greater than W_1^* and a magnetic-field discontinuity similar to that formed in a step junction [4] must also be considered. A superposition of these three equivalent circuits (see Fig. 2) has been assumed to approximate the total parasitics of the junction.

The swept frequency characteristics of the hybrid are shown in Fig. 3. Equal power levels were obtained at the two output ports [Fig. 3(a) and (b)] at both 4.1 and 8.2 GHz. The input-port return loss was greater than 22 dB [off the trace shown in Fig. 3(c)], while the isolated-port insertion loss was greater than 14 dB [Fig. 3(d)] at both frequencies. The dielectric and resistive losses were approximately 1 dB. These characteristics are adequate for balanced mixer applications, however, it may be possible to still further improve the performance by employing alternative geometries.

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The Characteristic Impedance of Rectangular Coaxial Line with Ratio 2:1 of Outer-to-Inner Conductor Side Length

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Recently, Riblet [1], [2] has given the exact dimensions of a family of rectangular coaxial lines with given impedance by conformal mapping. Before then the same problem was treated in [3], [4]. However, the previous literature does not include the case when the side of the outer and inner rectangle are in the ratio 2:1 both in width and in height: if in [1, eq. (11)] we put $\overline{OA} = \overline{DE}$ or $\overline{EO} = \overline{AB}$, modulus k coincides with modulus λ and the rectangular line becomes a square coaxial section, which is a special case of Bowman [5].

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It is the purpose of this letter to show a method for obtaining the characteristic impedance of rectangular coaxial lines with the ratio 2:1, both in width and in height, by the principle of conformal mapping.

We consider rectangular and circular regions [Fig. 1(a) and (b)]. We regard the center of each region as a source of lines of electric force, and regard the whole of the circumference of each region as a sink. In the circular region any radius coincides then with a line of electric force. In the rectangular region, however, it is not simple to draw exactly all of the lines of electric force. The four segments *OE, OF, OG,* and *OH* of Fig. 1(a) coincide with lines of electric force, and it is clear that these lines of electric force correspond to the same lines of the circular region. Strictly, the transformation

$$z = \frac{1 \pm \operatorname{cn}(Z,k)}{\operatorname{sn}(Z,k)}$$

maps the rectangle in the Z plane into the circle in the z plane, where

$$\frac{OH}{OG} = \frac{K'(k)}{K(k)} \tag{1}$$

$$\cos\alpha = k. \tag{2}$$

If we cut off a partial region $OFCG$ from both of the rectangular and circular regions along the lines of electric force OF and OG, then L-shaped region $ABFOGD$ corresponds to three quarters of the circle shown by the same letters. We transform the three-quarters circle in the z plane into a half-circle in the W plane [Fig. 1(c)] by the transformation

$$W = z^{2/3}$$

and we transform the W plane into a lower half w plane [Fig. 1(d)] by the transformation

$$w = \frac{1}{2}\left(W + \frac{1}{W}\right).$$

Then the half-plane capacity C is

$$C = \frac{K'(k_0)}{K(k_0)}$$

where

$$k_0 = \frac{\{1 - \cos(\pi/3 - 2\alpha/3)\}\{1 - \cos(2\alpha/3)\}}{\{1 + \cos(\pi/3 - 2\alpha/3)\}\{1 + \cos(2\alpha/3)\}}. \tag{3}$$

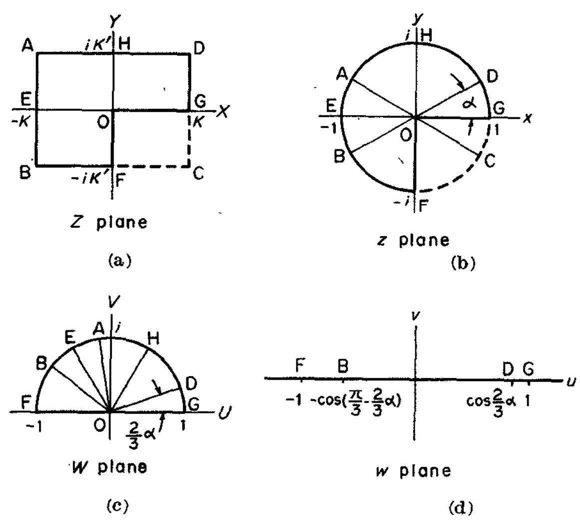

Fig. 1. Mapping of L-shaped region on lower half-plane.

The characteristic impedance Z_0 of the rectangular coaxial transmission line is then given by [1, eq. (13)], that is,

$$Z_0 = \frac{376.7K(k_0)}{4K'(k_0)} = 94.18\frac{K(k_0)}{K'(k_0)}. \quad (4)$$

Now, (4) determines k_0 corresponding to z_0 and sequentially, (3), (2), and (1) give α, k, and the shape of the rectangular coaxial line with the outer and inner ratio 2:1.

However, we cannot derive a rectangular coaxial line for any given impedance by this theory because the side-length ratio between outer and inner rectangles is restricted to 2:1. The characteristic impedance obtained by this method is restricted to the following:

$$0 \leq Z_0 \leq 36.81$$

where 36.81 is the value for the case when the coaxial line is square, though this value is 36.771 in [6, table I].

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A Waveguide Applicator for Sheet Materials

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***Abstract*—A partially loaded waveguide applicator for uniform heating of high-attenuation sheet materials is described. Experimental results are presented to exhibit its performance.**

INTRODUCTION

Microwave heating systems are finding greater acceptance as efficient economical means of industrial processing. One of the most important classes of industrial microwave systems concerns the heating of thin web or sheet materials. Various types of applicators have been designed to make efficient use of available power and heat the material uniformly. One of the earliest forms of such applicators is the serpentine or the meander-line applicator. Although quite efficient, it has problems of uniformity due to attenuation as well as due to standing waves in the system by reflections in the waveguide bends and at the edges of the web. Various schemes have been suggested to overcome these problems [1]–[5]. Most of these schemes become impractical when the attenuation of the web is very large. For such webs most of the microwave energy is attenuated before it reaches across the other end of the web, resulting in an extremely nonuniform heating. Use of tapered ridge waveguides has been suggested by Bleackely *et al.* [6] to overcome this problem. This letter presents another method of improving the uniformity by introduction of low-loss tapered dielectric slabs along the waveguide walls.

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THEORY

The basic principle involved is depicted in Fig. 1. It shows a rectangular waveguide partially loaded with two identical low-loss dielectric slabs. The introduction of the dielectric slabs tends to concentrate the field in the dielectric region and reduce the field in the middle of the waveguide. The reduction of the field in the center of the waveguide depends upon the dielectric constant, thickness, and position of the slabs. The theoretical analysis for such structures is well known and has been dealt with in detail by many authors [7], [8]. Some experimental results are presented in Table I to illustrate the effect. They were obtained by introducing slabs of materials having various dielectric constants and thicknesses into an X-band waveguide operating at a frequency of 9.9715 GHz. The results indicate that with appropriately shaped dielectric slabs one may be able to achieve a wide range of field intensity profiles along the length of a waveguide.

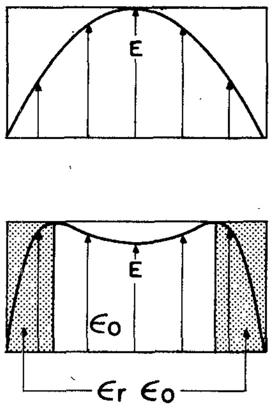

Fig. 1. Reduction in field strength of a waveguide due to dielectric loading.

TABLE I
REDUCTION IN FIELD STRENGTH OF AN X-BAND WAVEGUIDE FOR VARIOUS DIELECTRIC LOADINGS

DIELECTRIC	THICKNESS (INCHES)	INPUT VSWR	INSERTION LOSS (dB)	REDUCTION IN FIELD STRENGTH (dB)
	0.080	1.01	0.02	0.7
POLYSTYRENE	0.160	1.02	0.03	1.6
$\varepsilon_r' = 2.54$	0.240	1.02	0.45	3.3
	0.080	1.01	0.04	0.5
PLEXIGLASS	0.160	1.07	0.30	1.4
$\varepsilon_r' = 2.59$	0.240	1.02	0.85	3.6
	0.080	1.08	0.20	1.2
STYCAST	0.160	1.32	2.45	10-15
$\varepsilon_r' = 7.00$	0.240	2.35	3.85	18-19